Many studies have shown that the service behaviors of two-way slab systems are sensitive to the effects of construction loading. Because construction loads can approach or even exceed service level design loads, shoring and reshoring must be used to meet safety and serviceability requirements per ACI 318-08. Sustained, early-age loading can also have a significant impact on long-term deflections. Even if the shoring and reshoring schedules meet recommendations in ACI 347-04 or ACI 347.2R-05, unanticipated deflections can damage nonstructural items or require remedial measures.

This article provides a simplified calculation method for estimating deflections of two-way slabs in multi-story buildings. Early-age loading and its effects on concrete properties (elastic and creep behaviors) are incorporated. The method is implemented in a spreadsheet format, allowing users to investigate the effects of slab dimensions, design loads, and shoring/reshoring scenarios on long-term deflections. The method is shown to provide good agreement with data from laboratory tests of slabs.

**CONSTRUCTION LOADING**

For the common case where individual slabs in a shored/reshored system are of equal thickness, the load carried by a slab during construction \( w_{\text{const}} \) can be estimated as

\[
w_{\text{const}} = k_1 k_2 R w_{\text{slab}}
\]

where \( k_1 \) is a factor to allow for variations in slab stiffness, \( k_2 \) is a factor to account for weight of formwork, \( R \) is a factor (commonly called a load ratio) that varies with the number of levels of shoring and reshoring in the system, and \( w_{\text{slab}} \) is the self-weight of the slab; \( k_1 \) and \( k_2 \) typically have assigned values of 1.1.

An example of a load history for a slab with one level of shores and two levels of reshores is shown in Fig. 1. On removal of shoring at time \( t_s \), the slab carries \( w_{\text{slab}} \). Reshoring is then installed before the shoring for the next level is installed. When the new slab is cast at time \( t_{\text{start}} \), the slab in question is loaded by \( w_{\text{const}} \) per Eq. (1). At time \( t_{\text{end}} \), the load on the slab drops as the slab no longer supports reshores. The slab carries its own weight and any superimposed loads during the subsequent construction process. This load is termed \( w_{\text{sus}} \).

During the period \( t_{\text{start}} \) to \( t_{\text{end}} \), short-duration construction live loads will be applied to the hardened slab during the subsequent placements. These loads will increase the likelihood of cracking in the slab. At time \( t_{\text{inst}} \), nonstructural elements such as partition walls, doors, and windows are installed—adding dead load to the slabs. Between \( t_{\text{end}} \) and \( t_{\text{inst}} \), loads can include temporary storage of construction materials.
For most deflection calculations, the slab is assumed to carry its own weight, the design superimposed dead load, and a sustained portion of live load after tend. The sustained live load is usually taken as 10% of the design live load, and the remaining portion of live load can be applied at any time during the service life. For the spreadsheet described in this article, the live load is assumed to be applied as a short-term load at 5 years. For simplicity, the same level of sustained load is assumed to be present before and after the installation of nonstructural elements.

**DEFLECTION CALCULATIONS**

ACI 435R-95\textsuperscript{7} summarizes methods for calculating deflections of two-way slabs. These include methods based on the crossing beam analogy in which the mid-panel deflection is calculated as the sum of a unit-width column-strip deflection and a unit-width middle-strip deflection (Fig. 2). Results obtained using this analogy have been shown to compare favorably with those from finite element analyses.\textsuperscript{8,9} The instantaneous column- and middle-strip immediate deflections $\delta_c$ and $\delta_m$ can be calculated using standard beam deflection equations

$$\delta = \frac{K \cdot w \cdot \ell^4}{384 \cdot E_c \cdot I_c}$$  \hspace{1cm} (2)

where $K$ is a factor to account for span boundary conditions and can be taken as 1.4 for interior panels and 2.0 for exterior panels without edge beams, $w$ is a uniformly distributed load, $\ell$ is the clear span, $E_c$ is the modulus of elasticity of concrete, and $I_c$ is the effective moment of inertia. Additional time-dependent deflection under sustained load can be estimated by applying a long-term load multiplier $\lambda(t_i, t_f)$, based on the ACI 209 creep coefficient,\textsuperscript{10} a correction factor accounting for concrete age at loading,\textsuperscript{1} and the creep factor provided by ACI 318\textsuperscript{2}

$$\lambda(t_i, t_f) = \left(\frac{(t_f - t_i)^{0.5}}{(10 + (t_i - t_f)^{0.5})}\right) \times k_c \times \frac{\xi}{(1 + 50\rho')},$$  \hspace{1cm} (3)

where $t_f$ is the age of the concrete when load is applied, $t_i$ is the age of the concrete at which deflection is calculated, $\xi$ is a creep factor (generally taken as 2.0 unless specific information is available on the creep characteristics of the concrete), and $\rho'$ is the reinforcing ratio for the compression bars at midspan. The factor $k_c$ accounts for concrete age at loading\textsuperscript{10}

$$k_c = 2.3 t_f^{-0.25} \leq 1.75$$  \hspace{1cm} (4)

Generalizing the principle of creep superposition\textsuperscript{11} to the superposition of deflections due to a series of load increments, deflection at time $t_i$ can be calculated as

$$\Delta(t_i) = \frac{K \cdot \ell^4}{384 \cdot E_c} \sum_{j=1}^{j} \frac{\Delta w(t_j)}{E_c(t_j)} \left[1 + \lambda(t_i, t_j)\right]$$  \hspace{1cm} (5)

where $\Delta w(t_j)$ is an increment of load applied at time $t_j$, and $E_c$ is a function of $f'_c$, which is also a function of time.\textsuperscript{10} The modulus of elasticity at time $t_i$ is obtained from

$$E_c = 57,000 \sqrt{(f'_c/(at_i + b))} f'_c, \text{ psi (with } f'_c \text{ in psi) or}$$

$$E_c = 4700 \sqrt{(f'_c/(at_i + b))} f'_c, \text{ MPa (with } f'_c \text{ in MPa).}$$  \hspace{1cm} (6)

For $f'_c$ specified at 3 days, the coefficients $a = 0.63$ and $b = 1.11$ are used. For $f'_c$ specified at 28 days, the coefficients $a = 0.85$ and $b = 4$ are used.

Deflections can thus be computed at a series of times $t_i$ to provide the desired deflection-time history. The time-dependent modulus of elasticity is based on the compressive strength at the time of load application.

Bažant\textsuperscript{12} points out that the principle of superposition predicts too much recovery on unloading by as much as twice the observed amount. To account for the fact that creep deformations are not completely recoverable, a
factor of one-half is applied to the multiplier for load reductions (negative load increments).

For a uniformly distributed load, the moments in column strips are higher than the moments in middle strips. Based on the distributions of moments between column and middle strips prescribed in ACI 318, the uniformly distributed load $w_{UDL}$ is adjusted as follows for use in Eq. (5):

For the column-strip equivalent load

$$w_{col} = 1.35w_{UDL}$$  \hspace{1cm} (7)

and for the middle-strip equivalent load

$$w_{mid} = 0.65w_{UDL}$$  \hspace{1cm} (8)

For rectangular panels, a further adjustment is made to the middle-strip load to account for the fact that the middle strip in the short direction is wider than the column strip. The effective moment of inertia is used to account for cracking in the slab. As a first approximation, $I_e$ is respectively taken as 40% and 80% of the gross moment of inertia for the column strips and middle strips. These values reflect the fact that column strips with higher moment intensities, particularly in the vicinity of columns, are more likely to be cracked than middle strips. The approximate values are similar to the assumptions proposed by Rangan.13

**COMPARISON WITH MEASURED DEFLECTIONS**

To evaluate the validity of the proposed simplified method for estimating slab deflections under time-dependent loads, calculated values were compared with test results reported by Gilbert and Guo.14 For each slab, $f'_c$ was based on the reported 28-day values for $f'_c$. Long-term multipliers of 2.0 for Slab S1 and 2.5 for Slab S2 were used based on reported creep data. Calculated and measured deflection-time histories are shown in Fig. 3. The deflections are overestimated in the initial loading stages for Slab S1; however, the comparison between calculated and measured values improves at later age. For Slab S2, calculated values at early age are again slightly greater than measured values, although the measured values overtake the calculated values at about 60 days. A sudden decrease in measured deflection occurred at about 100 days due to unintended wetting of the slab surface caused by rain. Eventually, the measured deflections again overtake the calculated values.

Gilbert and Guo14 reported that Slab S1 was initially uncracked, whereas some shrinkage restraint cracking was evident in Slab S2 from the time of initial loading. In both cases, time-dependent cracking was observed and attributed to the development of shrinkage restraint stresses. To simplify the calculation procedure, cracking is assumed at all load stages, as outlined previously. This approximation appears to give reasonable results considering the extent of uncertainties involved regarding environmental conditions, material properties, and load history. We obtained similar levels of agreement for Slabs S3 through S7 in the test series.

**EXAMPLES TO ILLUSTRATE USE OF THE SPREADSHEET**

The spreadsheet is implemented with a 7-day cycle time. The concrete age at stripping can be taken as 1, 3, or 5 days. Construction loading is determined using Eq. (1), with $R$ assigned as 1.33 for one level of shores and two levels of reshores ($1S + 2R$), 1.25 for one level of shores and three levels of reshores ($1S + 3R$), and 1.20 for one level of shores and four levels of reshores ($1S + 4R$). The spreadsheet can be easily modified for other shoring/reshoring scenarios or for an arbitrary load history. The spreadsheet computes the deflection-time history for column-strip, middle-strip, and midpanel locations. An example of the output is shown in Fig. 4.
To illustrate the use of the spreadsheet, an example is presented of a typical interior panel of a flat plate. The analysis is performed for three construction loading cases: $1S + 2R$, $1S + 3R$, and $1S + 4R$. In each case, a 7-day cycle is assumed with stripping at 3 days. It was assumed that the concrete strength at 3 days was $f'c$ and the load during shoring and reshoring did not exceed the design dead plus live load. The age-at-loading correction factor (Eq. (4)) varied from 1.75 at 1 to 3 days to 1.0 at 28 days.

Analyses are also performed for Case $1S + 2R$ with a 7-day cycle and 1-, 3-, and 5-day stripping times. The results (Fig. 5) show that deflection is not very sensitive to the number of levels of reshores for the range considered but is sensitive to the age at stripping.

**CONCLUSION**

Slab deflections, as affected by the construction process (particularly for multi-story buildings), can have a significant impact on serviceability. Although it’s well known that slab deflections are highly variable and cannot be predicted very accurately, a reasonable estimate of expected time-dependent deflection magnitudes can be obtained using the described calculation procedure. The procedure takes into account the construction loading history, time-dependent concrete properties, and effects of cracking. The results can assist engineers in the evaluation of proposed shoring and reshoring schemes as well as the effect of slab thickness on time-dependent deflections. The spreadsheet can also be used at the design stage to give an indication of potential deflection problems related to the construction process and provide an opportunity to modify the design as needed.

The method has been shown to produce reasonable agreement with results from test slabs made with 28-day compressive strengths around 4000 to 6000 psi (28 to 41 MPa). For higher-strength concretes, the modulus of

**Will the spreadsheet results match those of structural design software?**

Probably not. The dead-load deflections calculated by structural design programs typically:

- Use a moment of inertia based on the effects of dead load alone, not dead plus construction load;
- Don’t include the effects of shrinkage and temperature cracking; and
- Don’t include the effects of early-age prolonged loading.

The spreadsheet is calibrated against measured actual deflections, and it includes the effects of construction loads, cracking due to shrinkage and temperature change, and early prolonged loadings. The latter consideration is particularly important, as shoring/reshoring effects are likely to last about 1 month.
elasticity versus time function and the long-term multiplier may require adjustment.

References


2. ACI Committee 318, “Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary,” American Concrete Institute, Farmington Hills, MI, 2008, 473 pp.


7. ACI Committee 435, “Control of Deflection in Concrete Structures (ACI 435R-95),” American Concrete Institute, Farmington Hills, MI, 1995, 89 pp.


Received and reviewed under Institute publication policies.

Will structural design software predict actual deflections?

Probably not. As the introduction to ACI 435R-95, “Control of Deflection in Concrete Structures,” states: …the magnitude of actual deflection in concrete structural elements, particularly in buildings, which are the emphasis and the intent of this Report, can only be estimated within a range of 20-40 percent accuracy.

It’s likely the structural engineer’s design program will underestimate the actual dead load deflection because it fails to account for cracking due to construction loads, cracking due to shrinkage and temperature change, or creep effects due to early and prolonged loadings.

It’s also important to note that measured elevations may not provide true indicators of load deflections. ACI 117 allows a ±3/4 in. (±19 mm) tolerance for the formed slab bottom surface and finished slab top surface, both to be measured when the forms and shores are still in place. An elevation that is 3/4 in. (19 mm) lower than the design elevation might simply be the result of an initially low (yet within tolerance) slab elevation. It’s also important to note that if the slab surface elevations were not taken when the forms and shores were in place, the datum for slab deflections should be the elevation of the slab surface at the columns, not the design elevation.
The practical problem: The 3- to 9-month dead load deflection in two-way slabs

The construction sequence for buildings with two-way slabs is generally as follows:

- Erect shores and forms;
- Place and cure the slab concrete;
- When a specified concrete strength is reached, remove shores and replace with reshores;
- Continue this process until the slab is no longer supported by reshores;
- Install mechanical, plumbing, electrical, masonry, and interior wall framing; and
- Finally, at 3 to 9 months, install cabinets, molding, and floor coverings.

When the final finish trades start their work, they sometimes find that the floors are sagging too much to allow a good fit of the interior finishes. The owner or construction manager may then claim that the problem stems from inappropriate construction practices and use this as justification for requiring the concrete contractor to provide supplemental fill to bring the slab surface closer to level. On one multi-story building, for which the second author was a consultant, the cost of extra fill approached $1 million—this is too large a cost to ignore.

Because of this, engineers, construction managers, and owners need to understand that if the dead-load deflection at the time of finish installation is controlled by tolerances for finishes, the tighter tolerances must be considered in the design, specification, and construction process.

Designing in accordance with building code requirements for deflection is not a solution to this issue. Table 9.5(c) in ACI 318-08 includes minimum thicknesses of two-way slabs without interior beams that can be used without providing deflection calculations. These thicknesses are intended to provide some control of deflection. The commentary states:

R9.5.3.2—The minimum thicknesses in Table 9.5(c) are those that have been developed through the years. Slabs conforming to those limits have not resulted in systematic problems related to stiffness for short- and long-term loads. These limits apply only to the domain of previous experience in loads, environment, materials, boundary conditions, and spans.

Based on the second author’s experience in the past 5 years, the owner’s, architect’s, or construction manager’s expectations for slab deflections have changed and the expected deflections may be outside the domain of previous experience.

ACI 318-08 also allows the designer to use a smaller value for the thickness than prescribed by Table 9.5(c) if deflection calculations are performed. Table 9.5(b) gives limitations for maximum permissible computed deflections based on two criteria:

- Immediate deflection due to live load; and
- That part of total deflection occurring after attachment of nonstructural elements.

There are no criteria for the $1 million issue—deflection prior to installation of the nonstructural elements. If this deflection is limited to a very small value by tight tolerances for finish work, the designer needs to take that into account. For instance, if a 10 in. (250 mm) thick slab is required by Table 9.5(c) and the designer chooses an 8 in. (200 mm) thick slab based on calculated deflection, the gross moment of inertia is about halved and the deflections may increase by a factor of two. Such additional deflection may meet the requirements in ACI 318-08 but still result in the need for costly repairs to allow the finishes to fit properly.

Andrew Scanlon, FAcI, is a Professor of civil engineering at The Pennsylvania State University, University Park, PA. He is Chair of ACI Committee 435, Deflection of Concrete Building Structures, and a member of ACI Committees 224, Cracking, and 348, Structural Reliability and Safety. His research interests include safety and serviceability of concrete structures.

Bruce A. Suprenant, FAcI, is President of Concrete Engineering Specialists, Boulder, CO. He is a member of ACI Committees 117, Tolerances; 222, Corrosion of Metals in Concrete; 228, Nondestructive Testing of Concrete; 301, Specifications for Concrete; and 302, Construction of Concrete Floors.